(Following Paper ID and Roll No. to be filled in your Answer Book)
PAPER ID: 9585 Roll No.

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2013-14

ELEMENTARY MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: - Attempt all questions.

SECTION-A

1. Attempt all parts of this question:

 $(10 \times 2 = 20)$

- (a) Evaluate $\lim_{x\to 3} \frac{x^2-9}{x-3}$.
- (b) Find the derivative of $y = 9x^2 + \frac{3}{x} + 5 \tan^{-1} x$ with respect to x.
- (c) Find the critical points of $f(x) = 9x^2 + 12x + 2$.
- (d) Find the slope of the tangent to the curve $y = x^3 x$ at x = 2.
- (e) Evaluate ∫cos³x e^{logsinx} dx.
- (f) Evaluate $\int_0^{\pi} e^{-2x} \sin x \, dx$.

(g) Find the order and degree of the differential equation

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} - 6 \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - 4y = 0.$$

- (h) Solve: $\frac{dy}{dx} = e^{x-y}$.
- (i) Two coins are tossed once. Find the probability of getting two heads.
- (j) If $P(A/B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{10}$, find P(B).

SECTION-B

- 2. Attempt any three parts of the following: $(3\times10=30)$
 - (a) Find $\frac{dy}{dx}$ if $y = \sin(\sqrt{\sin x + \cos x})$.
 - (b) The length of a rectangle x is decreasing at the rate of 3cm/minute and the width y is increasing at the rate of 2cm/minute. Find the rates of change of the area and perimeter of the rectangle if x = 10 cm and y = 6 cm.
 - (c) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.
 - (d) Solve: $(x+1)\frac{dy}{dx} y = e^x(x+1)^2$.

(e) An urn I contains 3 white and 4 red balls and urn II contains 5 white and 6 red balls. One ball is drawn at random from one of the urns and is found to be white. Find the probability that it was drawn from urn I.

SECTION-C

Note: Attempt any two parts from each question of this Section. $[(2\times5)\times5=50]$

- 3. (a) Differentiate $y = x^2e^x \sin x$.
 - (b) Evaluate $\lim_{x\to 0} \left(\frac{3^x 2^x}{x} \right)$.
 - (c) If $y = \left(\frac{1-\tan x}{1+\tan x}\right)$, show that $\frac{dy}{dx} = \frac{-2}{(1+\sin 2x)}$.
- 4. (a) Find f'(x) if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.
 - (b) Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where its cuts the x-axis.
 - (c) Find the approximate change in the volume V of a cube of side x meters caused by increasing the sides by 2%.
- 5. (a) Evaluate $\int \frac{(x-1)}{(x+1)(x-2)} dx$.

(b) Find the sum of the series as $n \to \infty$

$$\left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}\right]$$

- (c) Show that $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}.$
- 6. (a) Obtain the differential equation if $y = a \cos x + b \sin x$.
 - (b) Find the general solution of the differential equation $(1 + x^2) dy + (1 + y^2) dx = 0$.
 - (c) Solve the differential equation:

$$\frac{dy}{dx} = \frac{y}{x} + x \sin \frac{y}{x}$$
.

- (a) In a single draw of two dice. Find the probability of obtaining a total of 8.
 - (b) A bag contains 6 white, 5 black and 4 yellow balls. Find the probability of getting either a white or a black ball in a single draw.
 - (c) An urn contains 3 red and 4 black balls. Two balls are taken out at random. Find the probability that the balls are of different colours.